Sampling-Based Path Planning for Cooperative Autonomous Maritime Vehicles to Reduce Uncertainty in Range-Only Localization

Jonatan Scharff Willners, Lachlan Toohey, and Yvan Petillot

***Abstract*—The following letter presents an adaptive path planning algorithm for cooperative localization in the maritime environment. It considers the scenario where an Autonomous Surface Vehicle (ASV) acts as a Communication and Navigation Aid (CNA) to support Autonomous Underwater Vehicles (AUVs) with range measurements. As AUVs have no access to GPS while submerged, range measurements can bind the otherwise continuously growing navigational error. This can be done by methods such as range-only Extended Kalman Filter (EKF). In such methods, the resulting uncertainty and positional error depends on the geometry between transmitter (CNA) and receiver (AUV). This letter proposes a planning algorithm that combines priority based expansion of a search tree with random sampling-based exploration to position the CNA at strategic positions to transmit ranging messages at optimal times to reduce the uncertainty and error at the AUVs' position. The approach is validated and shows an increased confidence for AUVs' localization in simulated environments as well as real experiments using a dataset gathered from AUV Sirius.**

***Index Terms*—Acoustic communication, cooperating robots, localization, marine robotics, motion and path planning.**

I. Introduction

AUVs are seeing more use every year for missions including mine countermeasures, bathymetric surveys and search and rescue missions. The quality of the data gathered from such missions is related to how well it can be geotagged. An object of potential interest loses much of its value if the uncertainty in its position is too high to enable reacquisition in a subsequent mission. The collected data on an AUV is in most cases gathered while the vehicle is submerged, an environment which strongly attenuates electromagnetic waves. As such there is no access to GPS and the vehicles' main source of navigation is based on Dead Reckoning (DR). DR is performed by integration of inertial navigation data, which by nature is noisy and results in the vehicle's position estimate drifting over time without bound if no external reference is used. One option is vision or laser based Simultaneous Localisation And Mapping (SLAM) [1]. These methods do however rely on adequate water conditions, such as turbidity and illumination and having distinguishable features which are reobserved. A more generic approach is to use Acoustic Communication (ACOMMS) to triangulate or trilaterate a position based on communication with external sources. These external sources can be static beacons or mobile. Using static beacons limits the operational area to the acoustic range of the transponders and they are required to be manually deployed and retrieved, which can be a time consuming task. Alternatively by using ASVs in the operating area of the AUVs the vehicles can adapt to each other, and hence not limit the area of operation to a static region.

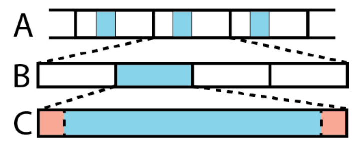

Fig. 1. TDMA is used to divide time among units to reduce message collision on a shared channel such as acoustics in water. **A**: Time is divided into frames. **B**: A frame consists of time slots. **C**: A time slot is the time a platform has the possibility to transmit and has an optional guard time in the beginning and end to avoid collisions from other time slots.

ACOMMS suffers from many limitations when compared to communication using electromagnetic waves. It has low bandwidth, high energy consumption, low propagation speed and operates on a shared communication channel. Due to these limitations, it is important for the operating vehicles to reduce message collisions to save energy and time. This can be done by implementing a protocol such as Time-Division Multiple Access (TDMA) (Fig. 1). For the TDMA protocol to be reliable long term, the platforms need to have synchronised clocks, which also enables measurement of ranges through One-Way-Travel-Time (OWTT) [2]. This is performed by including Time of Launch (ToL) in transmitted messages. The receiving platforms can then subtract the ToL from the Time of Arrival (ToA) and multiply by the speed of sound in water (approximately 1500 m/s). Other

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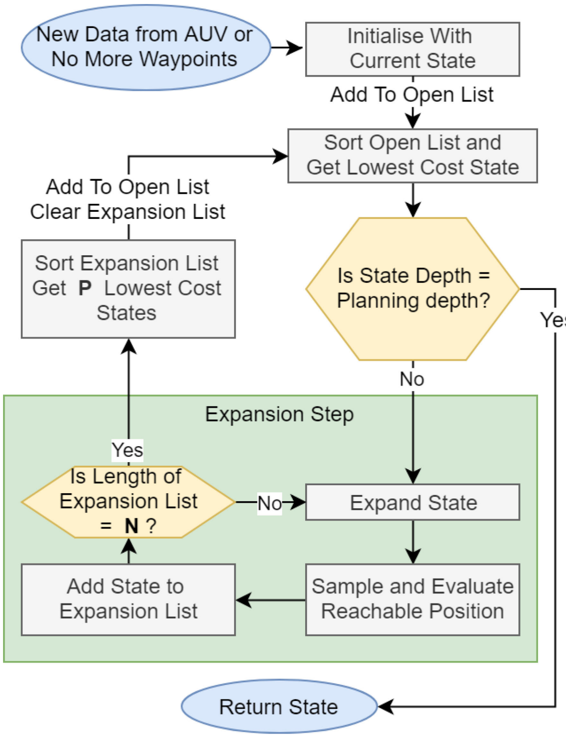


Fig. 2. The presented path planner is a combination of priority- and sampling-based methods to find a path consisting of a defined number of waypoints and optimal time for a CNA to transmit localisation messages to AUVs.

methods to measure distance and/or angles include Two-Way-Travel-Time (TWTT) or Ultra-Short Base-Line (USBL) [3]. However, these do not scale well with the number of platforms as they require one message exchange per vehicle. OWTT message can on the other hand support all receiving platforms with ranging measurement from the same message [4]. Time Difference of Arrival (TDoA) is another method that scales well with the number of platforms. This is mainly used for static sensor nodes [5] as the error will grow with the movement of the vehicle. By measuring the distance to a source with a known position, different methods can be used to estimate the receiver's position. This approach of localisation is referred to as trilateration or range-only localisation. There are multiple approaches [6]–[9] to solve this problem including Unscented Kalman Filter (UKF), Particle Filter (PF) and EKF or by solving the Non-linear Least Squares (NLS) problem. The accuracy of the result is however dependent on the geometrical relationship between the receiver and the source and will affect the uncertainty of the localisation method [10], as can be seen in Fig. 3.

This letter presents a path planner for an ASV to support AUVs with range-only localisation messages. The proposed approach applies a sampling-based expansion to a search tree, while choosing the optimal time of transmission for localisation messages. Optimal time in this letter refers to the time within a discrete set of times, in which a transmitted localisation message is estimated to be most efficient, as seen in Fig. 5b. Our approach is validated in simulations and with datasets collected on an AUV, showing a reduction in mean positional error for the AUV's in all evaluated scenarios compared to other approaches.

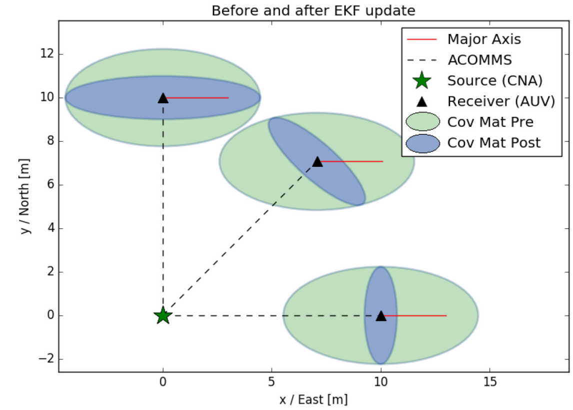
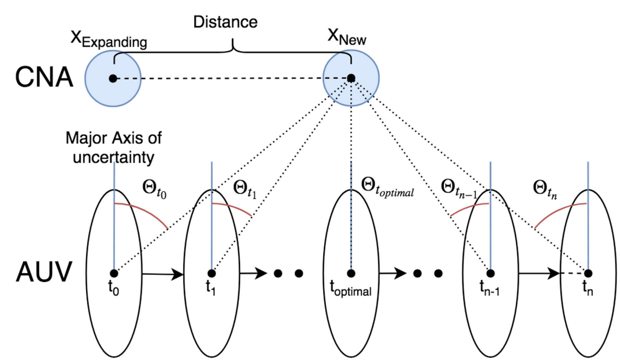


Fig. 3. The estimated uncertainty of the system is described by the covariance matrices which can be represented as ellipses, which here can be seen before and after a range-only EKF update on the receiver (AUV). The area of the ellipses is reduced by the most from the geometric relationship where the source (CNA) is on the major axis of uncertainty.



(a) A sampled position (X_{New}) is evaluated to find the optimal ToL which is the t that results in the lowest Θ_t. Θ_t is the residual angle between the AUV's major axis of uncertainty and the vector between the estimated positions of the vehicles (4) in the time window where the CNA is allowed to transmit.

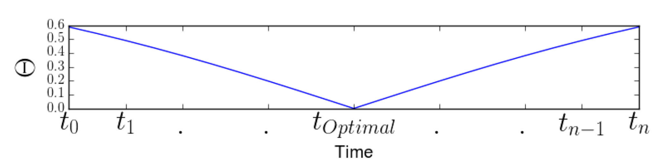
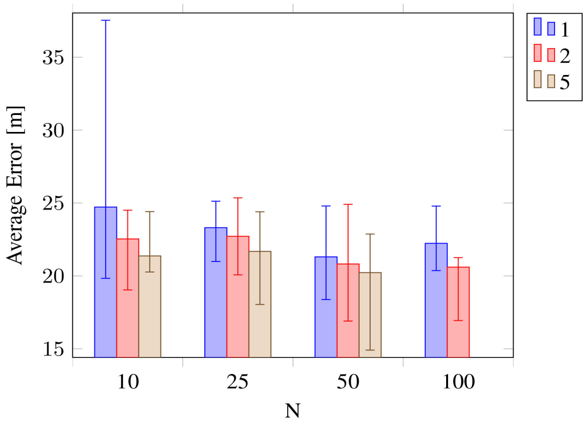


Fig. 4. The expansion is performed by randomly sampling a new position and then evaluating when, within the TDMA slot the optimal ToL for a ranging message would be.

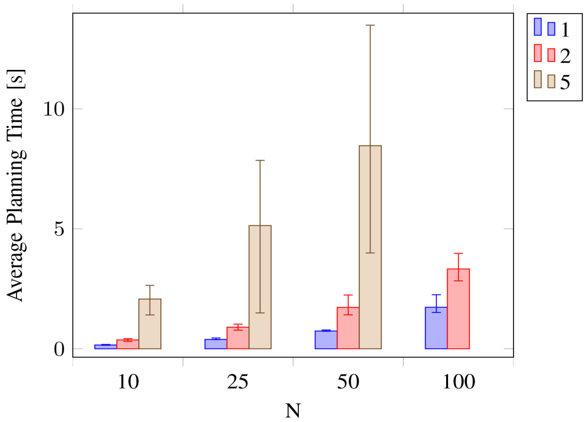
The remainder of the letter is organised as follows: Section II presents related work on how geometrical relationships affect range-only localisation and planning for cooperative localisation. The proposed algorithm along with the range-only EKF used is described in III. The results from the work are presented in Section IV and V concludes this letter.

II. Related Work

The problem of positioning a CNA at strategic positions to support AUVs with localisation through ACOMMS can be based either on a priori known information about the paths of the AUVs



(a) The average error on AUV Sirius.



(b) The planning time depends on the number of expansions and samples.

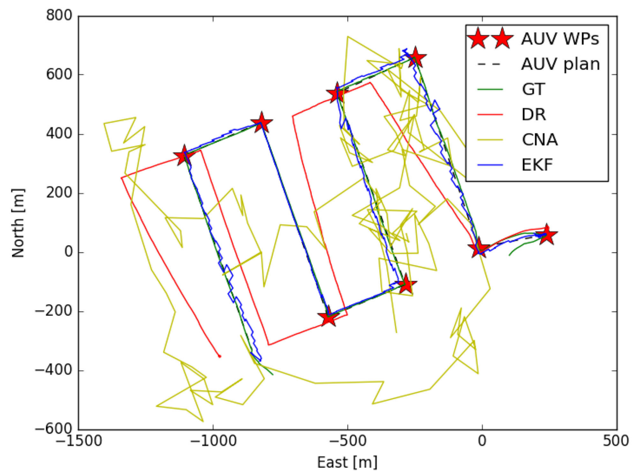
Fig. 5. The average error and execution time to plan next 4 waypoints (∼160 seconds), each set of parameters are simulated 10 times. The simulations use a navigational dataset collected on AUV Sirius combined with the proposed adaptive planning approach on a CNA. The different bars are the number of nodes saved from an exploration (The P value in the algorithms and flowchart), N is the number of samples during exploration. The average error on proposed approach is lower than the approaches it is compared to, as can be seen in Table II.

[11] or based on in-situ information received. The latter is in many regards more applicable, as the maritime environment is highly unpredictable and fixed plans are unlikely to succeed in real scenarios. Munafò *et al.* calculate the placement of surface vehicles as a response to incoming positioning data from submerged platforms [12]. This is performed by sampling on the perimeter of a circle and evaluating these points to select the most beneficial transmission position. The surface vehicle then sets the course towards the selected point. This is repeated whenever a positioning message is received from an AUV, but only plans one step ahead in time. Another next optimal step planner is presented by Bahr *et al.* where the surface vehicle samples all positions to build a cost map as a grid of discrete locations within an outer reachable region of the ASV [13]. The region is based on time to next transmission and the ASV's maximum speed. The location within the grid that has the greatest reward is chosen as the next waypoint. These two methods optimise only for the next step where a longer planning horizon may prevent the CNA falling behind or reduce uncertainty to a greater extent over the extended planning window. Walls and Eustice [14] plan a surface vessel with the help of a set of parameterised trajectory classes (zigzag and diamond) to update an AUV. The AUV's mission plan is assumed to be nominally known. Dynamic Programming [15] and Markov Decision Process [11] have been implemented to plan the path of a CNA to aim to transmit messages at the major axis of uncertainty of the AUV, showing reduced uncertainty compared to circular or static CNA trajectories. The CNA is encouraged to operate within a minimum and a maximum distance from the AUV to avoid collision and to stay within communication range by heavily penalising paths outside the acceptable boundaries. All vehicles travel at the same speed and the methods are used to control the heading of the CNA. Hudson *et al.* also applies penalties depending on inter vehicle distance while using look-ahead (planning a few steps ahead) and pre-decided trajectories to plan a path for the CNA to support multiple AUVs by adding the best of the trajectories to the search tree in each iteration [16]. The method can be used both with pre-planned missions on the AUVs or as a response to received acoustic messages. Other commonly mentioned approaches include planning the CNA with zigzag pattern [6], [13], diamond shaped paths [13] or circles [6], [15]. While these approaches are easy to implement, diamonds and circles patterns can result in a reduction of geometrical differences between measurements being small. If the pattern is too large, multiple consecutive signals origin can be close to on a straight line. If too small, the geometrical relationship will not differ much between signals to a vehicle far away the effect will be similar to using a static buoy. Zigzag patterns are simple to use for following a single AUV but might not be as useful for multi-AUV scenarios. Another drawback of zigzag is that the geometrical relationship can become a set of combinations, which might not reduce the uncertainty to the same extent as adaptive planning and positioning.

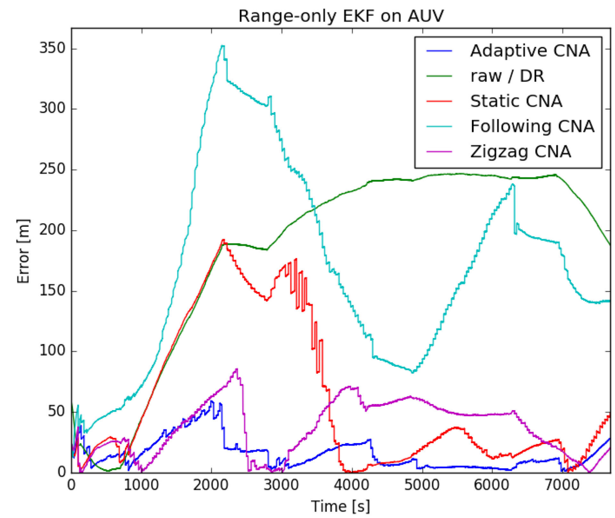
The work presented in this letter applies similar penalty functions as seen in [11], [15], [16] to discourage positions that are too close or far away from the AUVs. Unlike approaches presented in [12] and [13], where a one-step ahead planner is used, our approach uses sampling-based methods to enable multiple-step ahead planning and chooses the plan which leads to an overall reduction of AUVs uncertainty over time rather than over the next step alone.

III. Method

The algorithm presented in this letter is a spatio-temporal planner for CNA vehicles to find a path of waypoints, along with times, to transmit localisation messages from, to reduce uncertainty on receiving platforms. The approach combines priority based expansion of a search tree with a random sampling-based exploration. The exploration includes finding the ToL of localisation messages when the estimated geometrical relationship between vehicles should reduce the uncertainty the most. This geometrical relationship which reduces the uncertainty the most is when a message is transmitted from the semi-major axis of



(a) The adaptive path planning approach running with navigational dataset (DVL and orientation) collected by AUV Sirius outside Tasmania, Australia.



(b) The resulting error on AUV Sirius with a range-only EKF, the presented (adaptive) approach is compared to other approaches and DR.

Fig. 6. A dataset from AUV Sirius combined with a simulated CNA.

the ellipse which represents the uncertainty of the receiving vehicle's position [13], [15], which can be seen in Fig. 3. The choice to use a sampling-based approach for exploration is to reduce the number of states that are evaluated compared to a complete search approach as in [13]. Compared to [13] and [12], which plans for the next optimal position to transmit from, the proposed approach implements an iterative expansion of a search tree until a node in the tree at a specified depth ω is under expansion. The benefit of planning for multiple waypoints ahead in time is that planning for the next optimal step might not be good in the long term as the CNA might end up in configurations (such as falling behind) which might not be able to find good transmission positions. When a node is under expansion with the depth ω, by traversing from this node to the root of the tree, a list of ω waypoints can be extracted. The search tree expands based on priority, selecting and expanding the state with the current lowest cost, in a similar fashion to heuristic planners such as A* [17]. The cost is based on the estimated geometrical relationship between the CNA and the AUVs and a penalty function. The expansion of a state is sampling-based, which has been proved to be efficient by many methods such as Probabilistic Road maps [18] and Rapidly-exploring Random Tree [19]. A state represents the *position* of a waypoint, along with the *time* at that position which should reduce the uncertainty the most on the target AUVs and other information including: *Depth, Parent, Targets, Cost*. The flowchart of the planner can be seen in Fig. 2.

Compared to [11], [15], [16], the proposed approach does not assume constant speed, a discrete set of motions/headings or periodic transmission of localisation messages. Instead the planner finds the time when the transmission of a localisation should be most useful in the TDMA slots in which the vehicle is allowed to transmit from. It does not assume a constant speed not a discrete set of motions, instead it plans for waypoints within a reachable region based on the vehicle's maximum speed and the time is able to move until it latest has to transmit a message in the next TDMA slot. As such the proposed planner can be seen as a more high-level planning, based on waypoints, where as the other approaches are based on direct control of the vehicle's heading under constant speed.

A. *Range-Only Extended Kalman Filter*

The work in this letter consider the EKF as a range-only localisation filter as it is widely used and has proven effective in the underwater environment [8], [20], [21]. However, other filters such as Unscented Kalman Filter [22] or Particle Filter could be used, as long as they are able to estimate the uncertainty. The EKF is based on a prediction and update step. The prediction step predicts the new state of the vehicle and associated uncertainty based on its current state and a motion model of the vehicle. The update step combines the new measurements with the predicted state to produce a new state estimate. The combination is done through the Kalman gain and takes into account the uncertainty in the predicted state and the measurement noise. When the relationships between measurements and state and/or the motion model are nonlinear, the Extended version of the Kalman Filter is used by linearising the equations around their current estimate. The linearisation is done by using Jacobians, which for the system in this letter can be seen in (1), where C and X is the CNA's and the AUV's position. For a more in-depth description of EKF, see [23]. To plan multiple steps ahead in time, it is necessary to estimate how the ranging measurements would affect the AUVs. This is done by simulating the AUVs' range-only EKF along with their estimated position over time. To do this the AUVs need to update the CNA over ACOMMS periodically with information such as: estimated position, velocity, current waypoint or heading and a subset of the covariance matrix.

Algorithm 1: Sample-Based Transmission Waypoint Planner.

procedure Planner (X_0, ω, N, P)
  $Open = priority\_queue([X_0])$
  while $Open! = \emptyset$ **do**
    $X_{open} = Open.pop(0)$
    if $X_{open}^{depth} == \omega$ **then**
      return X_{open}
    $\hat{X} = Expansion(X_{open}, N, P, \Omega)$
    for each $x \in \hat{X}$ **do**
      $Open.add(x)$

Algorithm 2: Expansion Step.

procedure Expansion (X, N, P)
  $Y = priority\_queue()$
  $\Psi = X.sample(N)$
  for each $\psi \in \Psi$ **do**
    $ev = priority\_queue()$
    $T = TDMA(X, \psi)$
    for each $t \in T$ **do**
      $\sigma = \sum_{\tau}^{X^{Targets}} \zeta(\psi, \tau_t) + \delta(\psi^{pos}, \tau_t^{pos})$
      $ev.add((X, \psi_t^{cost=\sigma}))$
    $Y.add(ev[0])$
  return $Y[0 : P]$

Using this data, AUV_i's position at time *t* can be estimated as $(X_i(t) = [x_i(t), y_i(t)]^T)$. The growth of the covariance matrix can also be estimated based on what sensors the AUV is equipped with. Most AUVs are equipped with pressure sensors to obtain depth, and range-only localisation does not give any information about orientation. Therefore the filter only needs to estimate position and uncertainty in 2D (in the Northings(Y)-Eastings(X) plane parallel to the surface) and hence the covariance matrix can be limited to the size of 2x2.

$$H = \begin{bmatrix} \dfrac{(C_x - X_x)}{\sqrt{(C_x - X_x)^2 + (C_y - X_y)^2}} \\ \dfrac{(C_y - X_y)}{\sqrt{(C_x - X_x)^2 + (C_y - X_y)^2}} \end{bmatrix}^T \tag{1}$$

The approach uses the measurement step of range-only EKF to estimate the uncertainty on the target platform after a ranging update from the CNA. In Fig. 3, it can be seen how the range-only EKF estimates the uncertainty, and how it is affected based on the geometric relationship between source and receiver.

B. *Sample-Based Transmission Position Path Planner*

The proposed path planner is outlined in Algorithm 1 and Fig. 2. It is based on iterative expansion of a search tree. In each iteration the lowest cost state is selected for expansion. The expansion is performed until a state with a depth of ω is under expansion. When such a state is found it is considered to be the solution. When a state is expanded, its offsprings' cost are added with their parents. As a state can never have a lower cost than its parent, prioritising expansion of the lowest cost state at all times ensures that the first state fulfilling the termination condition (state's depth $= \omega$) is the best solution to be found under the conditions.

The algorithm is executed when new data from an AUV is received or if the current list of waypoints is empty. The inputs to the algorithm are:

- X_0 - The current state of the CNA and its knowledge about the AUVs.
- ω - Number of waypoints to plan for (depth).
- N - Number of new states to add to a node under expansion.
- P - Prune the list of N newly expanded states to only keep P.

C. *Expansion of States*

The expansion of a state can be seen in Algorithm 2 and Fig. 5a. The expansion is performed by sampling the set Ψ containing N new random positions from *X*. The region of sampling is dependent on the CNA's maximum speed ($CNA_{v\_\max}$), current time and latest ToL within the next TDMA slot. This ensures that the new states are reachable.

Below the functions and equations used in algorithm 2 are described,

- $TDMA(X, \psi)$: This function returns a set (T) of discrete (1 second interval($[t_0, t_1, \ldots, t_{n-1}, t_n]$)) times based on the earliest time (t_0) the CNA could transmit from X_{New}^{Pos}. t_0 and t_n is described in (2) and (3). $TDMA_{n+1}^0$ is the earliest time in the next TDMA slot that the CNA is allowed to transmit from. A visual representation of this can be seen in Fig. 5.

$$t_0 = \max\left(X_{expanding}^{ToL} + \frac{Distance}{CNA_{v\_\max}}, TDMA_{n+1}^0\right) \tag{2}$$

$$t_n = TDMA_{n+1}^0 + TDMA^{SlotTime} \tag{3}$$

- $\zeta(\psi, \tau_t)$ is the residual angle Θ (4), which is in the range $[0, \frac{\pi}{2}]$. θ is the dot product between the vector of the major axis of the ellipse which represents the estimated covariance matrix and the vector between position ψ and the estimated position of the target at time t, ($\overrightarrow{\psi\ \tau_t^{pos}}$).

$$\Theta = \min(\theta, ||\pi - \theta||) \tag{4}$$

- The sampling function $X.sample(N)$ returns a set of N new reachable states from the position of state X. The random sampled points are within the distance $t_n * CNA_{v\_\max}$
- The penalty function $\delta(\psi^{pos}, \tau_t^{pos})$ adds a penalty to a state if its waypoint is too close or too far from the AUV under evaluation. This is to reduce the risk of collisions between vehicles and aims to stay within communication range. The penalty is based on 4 different zones: *Critical*, *Risk*, safe (*None*) and if outside of communication range (*Comms*). This can be seen in Algorithm 3.

TABLE I
PLANNING PARAMETERS USED IN SIMULATIONS

Parameter	single	multi	Sirius
TDMA slot time	20	20	20
AUV speed [m/s]	1.5	1.0	0-0.652
P	3	5	5
N	100	50	25
ω	5	5	4
Parameters applied to all presented cases			
$\Lambda_{Critical,Risk,Comms,None}$	1.0, 0.5, 0.5, 0		
$\lambda_{Critical,Risk,Comms}$	50, 100, 250		
CNA max speed	3.0 m/s		
TDMA slots	2		

Algorithm 3: Inter Vehicle Distance Based Penalty.

procedure $\delta(\psi, \tau)$
    $\Delta = ||\psi - \tau||$
    if $\Delta < \lambda_{Critical}$ **then**
        return $\Lambda_{Critical}$
    else if $\Delta < \lambda_{Risk}$ **then**
        return Λ_{Risk}
    else if $\Delta > \lambda_{Comms}$ **then**
        return Λ_{Comms}
    else
        return Λ_{None}

After the N new states have been added to the list Y of sampled expansions, the list is pruned to only keep the P lowest cost new states. This is performed as to reduce the search space.

IV. RESULTS AND DISCUSSION

The described path planner has been implemented and evaluated in a simulated environment and combined with navigational data from the AUV Sirius [24]. It has demonstrated a decrease in uncertainty and error in position in all scenarios to all the methods it has been compared to. For all simulated measurements a Gaussian noise has been applied. The method has been compared to zigzag and following behaviours as well as static beacons. The parameters used for the planner in the various simulations and with the real dataset can be seen in Table I.

A. Single AUV

Fig. 7a shows the most common and simplest scenario, where one CNA supports a single AUV. The CNA has some knowledge about how the AUV's covariance matrix grows over time and integrates this information into the planning algorithm. In Fig. 7a a part of a simulated survey leg performed by the AUV can be seen along with the path planned by the CNA. The path positions the CNA at a risk free waypoint for transmission. How the adaptive path planned by the CNA performs compared to other methods can be seen in Fig. 7b. It shows that the proposed method results in a significant reduction in localisation error, area of uncertainty and the size of the major axis of uncertainty. The other methods transmit a localisation message periodically.

B. Multiple AUVs

The path planner was also simulated in scenarios where multiple AUVs perform surveys. In this scenario the CNA has no knowledge about the sensors on the AUVs, and cannot estimate the growth of their covariance matrices between predicted ranging updates. The path of the CNA and the AUVs can be seen in Fig. 7c and the resulting uncertainty and error can be seen in Fig. 7d. The proposed method reduces the uncertainty in both AUVs compared to the other approaches. The method has been simulated with more AUVs in different configurations, showing similar results. In multi-vehicle scenarios, the vectors representing the major axis of uncertainty might not intersect, or might be far away. Hence the cost function is the sum of the residual angle and penalty function for all vehicles. Instead of aiming to minimise the error on one AUV the planner strives to minimise the sum of uncertainty on all participating vehicles.

C. Dataset - Sirius AUV

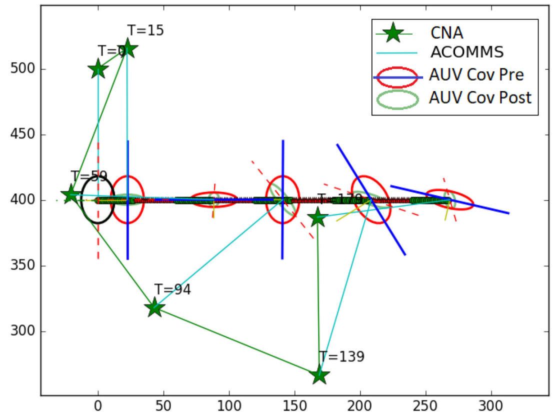
A dataset from Sirius AUV performing surveys outside Tasmania have been used to validate the effect of transmission from the planned waypoints. The data used for the EKF on the AUV integrates orientation and velocity from a Doppler Velocity Log (DVL) along with the simulated ranging measurements from the CNA. The AUV's ground truth is considered to be the localisation obtained from visual SLAM by Mahon *et al.* [25] combined with USBL. The paths of both vehicles can be seen in Fig. 6a and the resulting error in localisation on the AUV is compared to DR, a static beacon, a CNA following the AUV (with same speed and heading) and a CNA moving in a zigzag pattern can be seen in Fig. 6b. The AUV transmits updates to the CNA with its estimated position, heading, velocity and 2-by-2 covariance matrix periodically every 160 seconds. The mean error for the proposed adaptive method compared to others can be seen in Table II and in Fig. 5a, where the average error of the proposed methods is reduced for all simulations compared to the other methods. The simulations were performed using Python 2.7 on an i7-7820 (2.9 GHz) with 16 GB RAM.

D. Computational Efficiency vs Optimallity

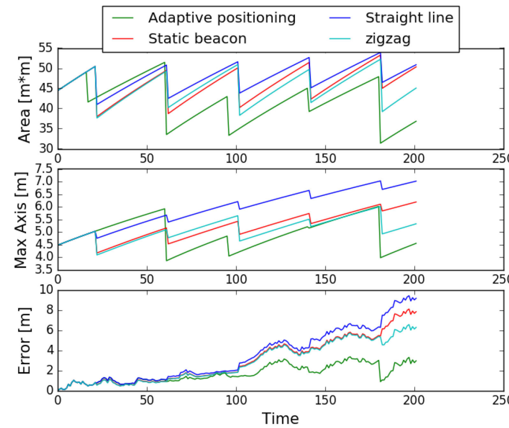
The choice of using elements from sampling-based path planning is to reduce the search space enough to run the algorithm in real-time. While it is possible to expand a complete search tree to find the optimal solution, which is estimated to reduce the uncertainty the most, all reachable lattice points with all possible (whole seconds) ToL have to be considered in the search space. One iteration with a maximum reachable distance r metres includes all reachable lattice points as seen in (5) [26].

$$N(r) = 1 + 4 * r + 4 \sum_{i=1}^{r} (\sqrt{r^2 - i^2}) \tag{5}$$

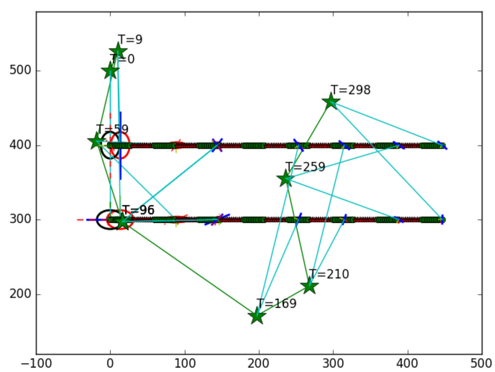
By extending (5) to include all possible ToL at respective lattice points it results in all possible states for each iteration to achieve the optimal position and transmission time. Each iteration expands the search tree by $O(T)$, where T is the time until latest possible ToL, nodes as seen in (5), to plan an optimal



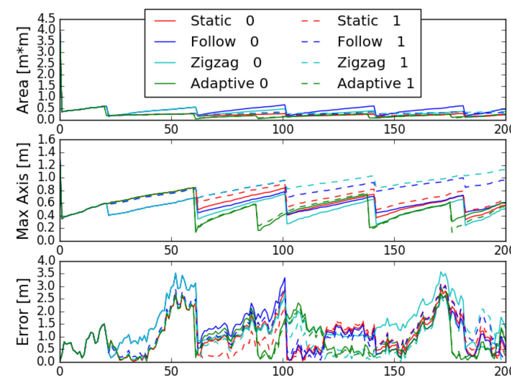
(a) The path planned for a CNA supporting 1 AUV.



(b) Navigation error (bottom) alongside error estimates (area of covariance matrix and maximum axis of covariance matrix) on an AUV using range-only EKF.



(c) The path on a CNA supporting 2 AUVs.



(d) The uncertainty and error on vehicle 0 and 1.

Fig. 7. Two scenarios (Top: 1 CNA - 1 AUV and bot 1 CNA - 2 AUVs) where the result on AUVs of the proposed approach is compared to a static beacon. The parameters of the scenario can be seen in Table I. In (a) and (c), the stars are the waypoints for the CNA, the ellipses are the estimated covariance, the blue line is the semi-major axis of uncertainty and the light blue line shows the communication to each vehicle at that position. The black ellipses represents the initial uncertainty.

TABLE II
COMPARISON OF THE MEAN ERROR IN METERS ON AN AUV BETWEEN PROPOSED ADAPTIVE PATH PLANNER (SEE FIG. 5A FOR FURTHER DETAILS) AND OTHER APPROACHES BASED ON A DATASET FROM AUV SIRIUS. THE PROPOSED ADAPTIVE APPROACH REDUCES THE ERROR SIGNIFICANTLY BY PLANNING A PATH THAT AIMS TO TRANSMIT MESSAGES FROM A POSITION WHICH IS ESTIMATED TO BE CLOSE TO THE MAJOR AXIS OF UNCERTAINTY ON THE AUVS

Method	Average Error [m]
DR	183.03
Follow	168.63
Static	60.34
Zigzag	36.49
Adaptive	21.95

path of M waypoints the number of expanded nodes would be $O(T)^M$, for a maximum speed of 1 m/s.

$$O(T) = \sum_{t=1}^{T} (T-t) * N(T-t) \tag{6}$$

$O(T)$ grows exponentially, while the proposed algorithm explores between N and $N * T$ and expands the search tree with the P most promising states each iteration. As such to evaluate all possible paths to find the optimal solution would be of a many orders magnitude larger than the proposed approach. This reduction of the search space enables real-time computation, and as seen in Fig. 5a and 5b, the larger P and N, the better solution, at the cost of longer execution time.

V. CONCLUSION AND FUTURE WORK

This letter has presented a novel adaptive path planning algorithm for a Communication and Navigation Aid which aims to reduce uncertainty on submerged Autonomous Underwater Vehicles. The approach combines a priority based search tree with sampling-based expansion to find strategic waypoints along with optimal times for transmission of ranging messages. Ranging messages can be used by methods such as range-only Extended Kalman Filter to bound the submerged vehicles' otherwise continuously growing positional error and uncertainty. The algorithm prioritises expansion of states in the search tree that minimises the angle between the vehicles and the major axis of uncertainty, which is the geometry that minimises the uncertainty. The planner adds penalty to states that have a higher risk of collision between vehicles or are outside of the estimated communication range. The algorithm takes Time-Division Multiple Access into consideration to reduce the risk of message collision on the shared acoustic communication channel. The approach has been verified and evaluated in a simulated environment combined with dataset from an AUV. It shows reduction in positional error and uncertainty on the AUVs to compared methods. The usage of the combination of priority- and sampling-based expansion of a search tree reduces the search space to allow online planning.

Future Work

In the future more vehicle and environmental dynamics are to be integrated to improve the sampling region of the planning algorithm. Another extension is to extend the algorithm to handle multiple CNAs. Estimation of the covariance matrices growth on the AUVs with unknown sensors based on the information transmitted to the CNA could increase the efficiency of the planner in long-term missions. For scenarios with multiple AUVs, adding weights to the cost function to prioritise the vehicles with the most uncertainty could decrease the error on the most uncertain vehicle. To find the optimal ToL, a search strategy such as gradient search could be applied.

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