Improving Acoustic Range-Only Localisation by Selection of Transmission Time

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***Abstract*—This work looks at when to transmit localisation messages in the scenario where an Autonomous Surface Vehicle (ASV) has a planned path and can estimate a submerged AUV's positions based on acoustic updates. Acoustic localisation messages are in general transmitted periodically, but by estimating the movement of the involved vehicles, messages can be transmitted at a time when the geometrical relationship between vehicles should reduce the receiving vehicles error and uncertainty to the greatest extent. This paper looks at how selection of Time of Launch (ToL) within a Time-Division Multiple Access slot can reduce this based on history of transmission and the estimated geometrical relationship between vehicles over time. The method to select ToL is dependent on the localisation method, we look at this from the perspective of Extended Kalman Filter and to solve the trilateration problem using Non-linear Least Squares. The benefit of proposed approach is in scenarios where the operating vehicles have their own objectives, and cannot adapt their path to achieve a more beneficial transmission position. The proposed approach shows a reduction of error and uncertainty, while using a navigational dataset collected by an Autonomous Underwater Vehicle when compared to other methods of which transmission times are selected.**

Keywords*—*ASV, AUV, Range-Only, Acoustic Localisation

I. Introduction

Maritime robots are seeing an increase in usage in various fields including Mine Countermeasures, archaeology, bathymetric surveys and pipe inspection among others. Conventionally these missions have often been carried out by a single Autonomous Underwater Vehicle (AUV), but as the vehicles and systems mature, multi-vehicle or swarm missions are seeing and increased usage. For most missions, if gathered data cannot be geotagged with high confidence the data becomes less reliable and useful. This is an issue for submerged vehicles as Radio Frequency (RF) signals do not propagate well in water and therefore, there is no access to Global Positioning System (GPS). Instead submerged vehicles, as AUVs, have to rely on Dead Reckoning (DR) from integration of sensor data to estimate its position. Due to noise and drift in sensors, this method results in an error which will grow continuously. If the estimated error grows too large, the most common approach is for a vehicle to surface to regain access to GPS, which is a time and energy consuming task. Due to this, it is desirable to use external sources as a way to localise the vehicle. This can be either by using features in the natural environment to perform e.g. Simultaneous Localisation And Mapping (SLAM) [1], [2] or artificial sources such as acoustic communication. Acoustic communication can be used in various ways as a method for localisation including measurement of the angle of the received signal (Ultra-Short Base-Line (USBL)), Time Difference of Arrival and range-only (distance travelled of an acoustic message). In this paper the latter is considered as it scales well with an increasing number of vehicles if One-Way-Travel-Time (OWTT) is used [3] to measure the distance travelled of the message. OWTT can be used if vehicles have synchronised clocks [4]. OWTT is based on messages that are transmitted with a time stamp of the ToL. When such message is received, the receiver can calculate the time difference of ToL and the time the message is received. By multiplying this value with the speed of sound in water, generally considered to be 1500 ms, the speed is however dependent on environmental factors such as conductivity, temperature, and pressure [5]. If the distance of an acoustic message is measurable and it contains data which includes the origin (transmission position) of the message, range-only localisation can be applied [6], [7]. There are different methods to solve this localisation problem such as recursive Bayesian methods, Monte Carlo [8] and numerical solutions. We will look at this from the perspective of Extended Kalman Filter (EKF) and by solving the trilateration problem using Non-linear Least Squares (NLS). In both of these methods, the quality of the solution is dependent on the geometrical configuration between transmitter and receiver. The problem of improving acoustic localisation is generally looked at as a path planning problem, where a dedicated vehicle can move to transmit from positions that are estimated to have a good impact on the receiving system [9]–[11]. For AUVs using EKF as a distance-based localisation method, Bahr *et al.* [10] choose transmission position based on that the uncertainty is decreased the most, if transmitted along the semi-major axis of uncertainty of the receiving vehicle [12], which can be seen in Fig. 2. Tan *et al.* [13] aims to transmit the first message along the semi-major axis and then plan paths for a vehicle to send consecutive messages with as close to 90° as possible to the receiving vehicle. In [14] it is stated that one of the main factors which reduce the performance in range-only localisation is "ranging from the same relative direction", which can be seen in Fig. 3a where the uncertainty is only reduced along one axis when a vehicle consecutively transmits on the same axis. Localisation can also be performed by solving the trilateration problem, which is to find the intersection between geometric shapes [15]–[17]. The quality of the solution in the trilateration problem is dependant on how the relative direction of incoming transmissions are

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distributed. In [18], Kelly *et al.* shows that the more evenly spread around the receiver the relative transmission positions are, the better the solution should be. Which is also shown in [19], where the quality of trilateration is evaluated based on landmark positions and their confidence. This can be seen in Fig. 5.

This paper aims to improve the quality of the range-only localisation by selecting when to transmit a localisation message, within a Time-Division Multiple Access (TDMA) slot (Fig. 1), such that transmitted message should reduce the error and uncertainty the most. It looks at the scenario where multiple vehicles have their own objective and therefore cannot re-plan their path to achieve a good geometrical relationship between the vehicles. This is performed by discretising the estimated paths of the vehicles and based on localisation methods minimising a cost function to find the estimated best ToL.

The remainder of the paper is organised as follows: section II presents the range-only localisation methods considered in this paper and III describes how the selection of ToL is performed to reduce the error and uncertainty on receiving vehicles. The results are presented in section IV and V concludes this paper.

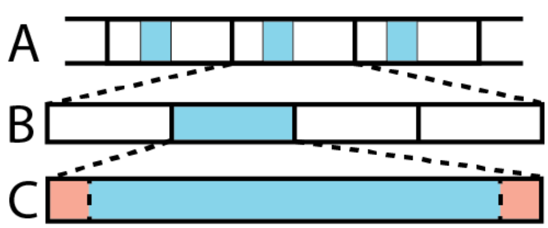


Fig. 1: TDMA is used to divide time among units to reduce message collision on a shared channel such as acoustics in water. **A**: Time is divided into frames. **B**: A frame consists of time slots. **C**: A time slot is the time a platform has the possibility to transmit and has an optional guard time in the beginning and end to avoid collisions from other time slots.

II. Range-Only Localisation

Range-only localisation is the method of estimating a position based on distance to one or more landmarks. Landmarks in this case is considered to be the transmission of an acoustic massage. For AUVs this is usually based by receiving an acoustic message containing the message's origin and time-stamp(s) to calculate to distance the message has travelled. While there are many methods to perform this type of localisation, this paper considers EKF and solving the trilateration problem using NLS. The uncertainty and error from range-only localisation is a product of the geometrical relationship between transmitter and receiver, the DR of the receiver and the error in the distance measurement. In this section the two methods are described along with how the error and uncertainty is affected by the geometrical configuration.

A. Extended Kalman Filter

The EKF is based on a prediction and update step. Each step estimates the state of the filter and the associated uncertainty (covariance matrix). The prediction step estimates the system's state and how the uncertainty changes based on the estimated noise of equipped sensors and the motion model of the vehicle. The update step tries to fit a measurement (range-only localisation message) to the predicted state of the system to update the estimated state and covariance matrix (2) according to how well the measurement fits the model. This is performed using the Kalman gain (1), which takes the uncertainty of the predicted state and measurement noise into consideration. While the measurements are non-linear, they can be treated as linear around the current estimate. The linearisation is performed through using a Jacobian (3). In (3), LM is the origin of the transmitted message and X the estimated state of the AUV. For an in-depth description of EKF, see [20].

$$K_k = P_k^- H^T (H P_k^- H^T)^{-1} \tag{1}$$

$$P_k = (I - K_k H) P_k^- \tag{2}$$

$$H = \begin{bmatrix} \dfrac{(LM_x - X_x)}{\sqrt{(LM_x - X_x)^2 + (LM_y - X_y)^2}} \\ \dfrac{(LM_y - X_y)}{\sqrt{(LM_x - X_x)^2 + (LM_y - X_y)^2}} \end{bmatrix}^T \tag{3}$$

The localisation filter on the AUV is assumed to be in the 2D plane (north (x) and east(y) parallel to the surface of the sea) as depth is considered known due to pressure sensors. The covariance matrix can be visualised as an ellipse in the 2D plane. To reduce the covariance matrix the most, a measurement should be observed on the semi-major axis of the ellipse representing the uncertainty [10], [13]. This can be seen in Fig. 2. If a measurement is on the semi-minor axis instead, the uncertainty is reduced the least. This can be seen in Fig. 3a where a vehicle transmits localisation messages in a zigzag pattern, but each message is transmitted from the semi-minor axis of uncertainty on the AUV. As such, the uncertainty is only reduced on one axis, while the other axis (along travel direction) grows unbounded (as in Fig. 3b).

B. Trilateration using Non-linear Least Squares

Trilateration is the problem to try to find the intersection between geometrical shapes. This can be numerically solved by for example NLS. NLS for trilateration is used to find the parameter (position/intersection) which minimises the difference between measured and the estimated distances (5). This is performed by trying to fit m observation to a non-linear model of n parameter, where $m > n$. For the range-only localisation problem, observations are the distances measured to their positional origins and the parameters is the intersection of those measurement. The reason that we need m to be larger than n is to find a unique solution, as the intersection of m shapes, where $m \leq n$ in n dimensions will produce n or more solutions. As in the EKF method, the trilateration problem is solved in 2D, which will be to solve the intersection of $m > 2$ circles. This paper will consider $m = 3$ measurements for the problem, as this is the least amount needed. While solving the

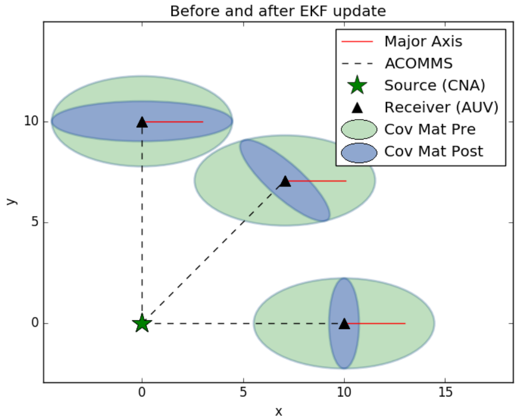
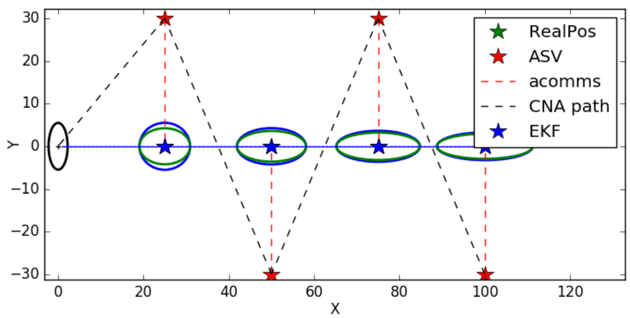
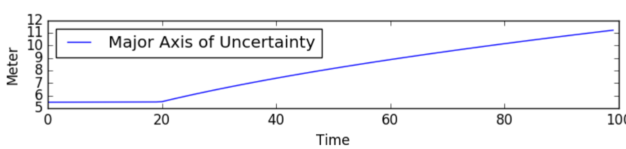


Fig. 2: Ellipses representing the covariance matrix before and after an EKF update. The geometrical relationship effects the resulting covariance matrix. The closer to the semi-major axis of uncertainty the landmark (source) is, the more the uncertainty is reduced.



(a) A zigzag pattern with periodical transmissions might in worst case only reduce the uncertainty on one axis. The black ellipse is the initial uncertainty (from the covariance matrix), the blue is the covariance matrix when an observation of a landmark is made, the green is after it have been incorporated in the EKF.



(b) The major axis of uncertainty grows unbounded when the observed landmarks does not change geometrical relationship. From the ellipse in Fig. 3a.

Fig. 3: An example where a zigzag pattern with periodical ToL minimises one axis of uncertainty while the other grows without a bound.

trilateration problem the measurements need to be considered to be received at the same point in time. This can be done by compensating the origins of previously received messages based on the DR of the vehicle since the time each message was received. A visualisation of this can be seen in Fig. 4.

$$\hat{r}_i = \sqrt{(x - x_i)^2 + (y - y_i)^2} \tag{4}$$

$$x, y = argmin(\sum_{i=0}^{n}(r_i - \hat{r}_i)^2) \tag{5}$$

The uncertainty, or the region of possible solutions, is dependant on the noise in the measurement of the distance of the signal, the DR of the vehicle between measurements and the distribution of the origin of those signals. How the distance and distribution of the landmarks affect the solution can be seen in Fig. 5. In Fig. 5b, it can be seen that when the landmarks are evenly distributed, the area of possible solutions is the smallest, which should produce the most accurate solution.

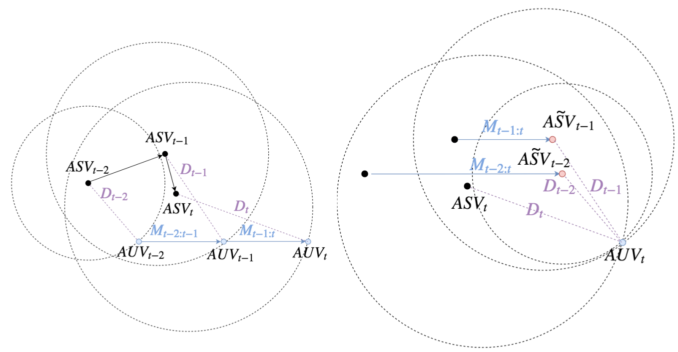


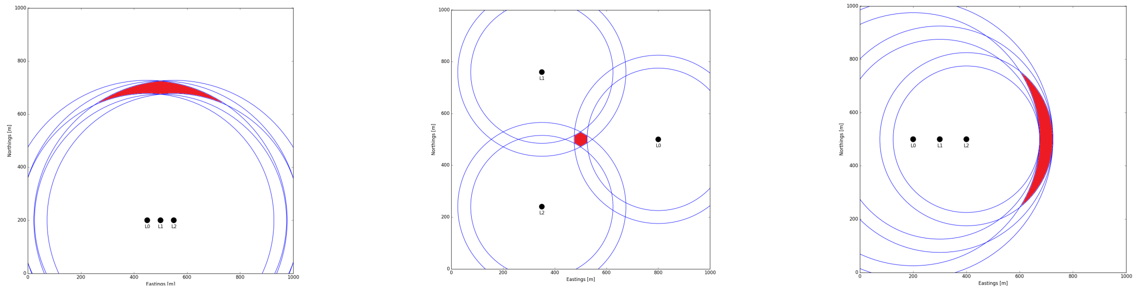
Fig. 4: By compensating the movement of the landmarks (CNA's transmission position), the AUV can solve the NLS problem as if all signals where received simultaneously.

III. Selection of ToL to Reduce Error and Uncertainty in Range-Only Localisation

The procedure of choosing the estimated best ToL is performed by creating sections containing a set of the estimated geometrical configurations between the vehicles over time. The number of configurations depends on the time available in the TDMA slot and the discrete time-interval this slot is divided into. The geometrical configurations are based on the estimation of the AUV's position (from data received over acoustic communication such as estimated position, heading and speed) and the pre-planned path of the transmitting vehicle. The sections, in which the transmitter is allowed to transmit from (according to TDMA), is used as the input to the algorithm which selects one ToL from each TDMA slot. An overview of this can be seen in Fig. 6. The selection of ToL is dependent on which localisation method is used, which will be described in this section.

A. Extended Kalman Filter

As mentioned in section I and II-A, the reduction of the uncertainty is greater the closer the transmission position is to a point on the semi-major axis of uncertainty [10], [13]. The method to choose ToL is therefore to aim to reduce the residual angle of the vector between receiver and transmitter and the semi-major axis. When planning for multiple transmissions ahead in time, the planner will estimate how a transmission of a localisation message will affect the receiving platform and update the model of the AUV based on this. In the case



(a) Small distribution - Large area of possible solutions

(b) Equal distribution - Small area of possible solutions

(c) Small distribution - Large area of possible solutions

Fig. 5: The distribution of the landmarks and the noise in the range measurement affect the uncertainty on the receiver. The more equal the landmarks are distributed the less uncertainty in the computed solution (red area).

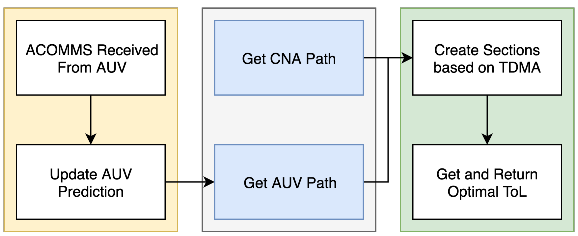


Fig. 6: The estimated path of the AUV is updated over acoustic communication. Using this information and the planned path of the CNA, the CNA selects the ToLs which should reduce the uncertainty and error on the AUV when a localisation message is transmitted at selected ToLs.

that some, or all, of the navigational sensors are known on the AUV, this can be used to estimate how the uncertainty would grow in the prediction step. This could be useful in scenarios where the AUV for is not equipped with a Doppler Velocity Logs (DVL) to measure speed and the uncertainty along the direction travelled is increasing more than the uncertainty based on the heading.

B. Tilateration using Non-linear Least Squares

Trilateration is based on solving the problem with a set of observations, as such, if localisation messages have been transmitted earlier than the the current instance of the ToL planner is executed, the information about the previous transmission should be taken into consideration. For the problem of solving for 3 measurements, the goal is to find a configuration in the sets such that the 3 angles (relative direction) from the AUV to the CNA is as evenly distributed as possible [18], [19]. This is done by finding the combination where a value from each of the sets A, B and C which minimises the function (8). However, after the first 3 sets have been evaluated and the fourth or later ToL is to be chosen, the history of planned ToLs should be used as an input. The input will then be that set A and B only contains 1 value (the previously selected ToLs), and as such the only value to be selected is the one from set C.

$$cost(\delta, \zeta, \theta) = CW(\delta, \zeta, \theta) + CCW(\delta, \zeta, \theta) \tag{6}$$

$$\begin{aligned} f(\alpha, \beta, \gamma) = &|cost(\alpha, \beta, \gamma) - cost(\beta, \alpha, \gamma)| \\ &+|cost(\beta, \alpha, \gamma) - cost(\gamma, \alpha, \beta)| \\ &+|cost(\gamma, \alpha, \beta) - cost(\alpha, \beta, \gamma)| \end{aligned} \tag{7}$$

$$a, b, c = min(f(\alpha, \beta, \gamma)\forall_{\alpha \in A, \beta \in B, \gamma \in C}) \tag{8}$$

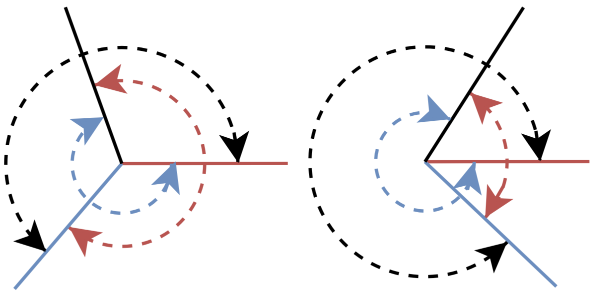
In (6), *CW()* and *CCW()* returns the minimum residual angle from θ in the clockwise respective counter-clockwise direction to ϵ and β. In (7), $|g(...) - g(...)|$ will have a cost of 0 if all angles are evenly distributed ($2\pi/3$ between each, as seen to the left in Fig. 7a) and a cost of 2π when all the angles are equal to each other. (7) adds a cost to each combination of the cost of the 3 angles. An example of the cost in different configurations of angles can be seen in Table I. A visual example where a few different sets of angles are evaluated to find the configuration (1 from each set) that minimises (8) can be seen in Fig 7b.

α	β	γ	cost(α, β, γ)
0	$2\pi/3$	$-2\pi/3$	0
0	$\pi/2$	$-\pi/2$	π
0	$\pi/6$	$-\pi/2$	$7\pi/3$
0	$\pi/6$	$-\pi/6$	3π
0	0	0	6π

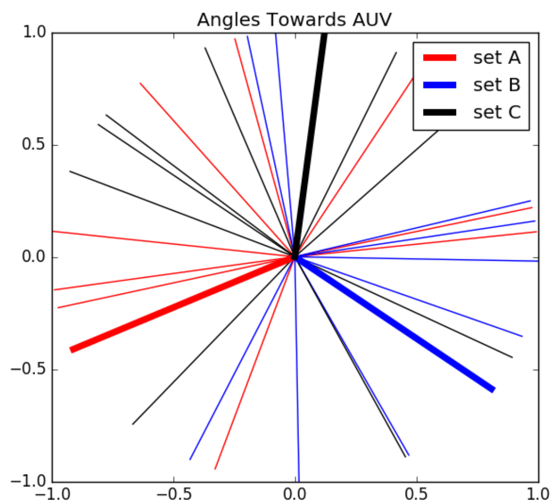
TABLE I: Example of the cost (equation (7)) between the three angles α, β and γ

IV. Results

The proposed method to select ToL was compared to the option of static transmission times in the TDMA slot. For the comparison a TDMA with a considered slot length of 20 seconds and two frames, (Fig. 1). The AUV is considered to update the transmitter every 4:th frame (160 seconds period). For the comparison, both moving transmitters (ASVs) and static (buoys) are considered with different objectives, paths and configurations. The resulting error and uncertainty on an AUV is compared in Table II and III. The AUV is based on a navigational dataset collected by AUV Sirius [21], which



(a) The function used to find optimal transmission positions for NLS compares the clockwise and counter-clockwise residual to the other evaluated angles which the estimated positions of the vehicles would create. (Left) evenly distributed angles creates good conditions to solve the NLS problem, while (right) is a worse setup to solve the problem, this can be seen in Fig. 5.



(b) 3 Sets of the angels from geometrical configurations between an AUV and an ASV. The thicker lines show the 3 configurations which creates the most evenly distribution between landmarks, and hence should produce the most accurate result.

Fig. 7: The geometrical relationship that produces the lowest error for the trilateration problem is when the angle (relative direction) of the received messages' positional origin are as evenly distributed around the receiver as possible, as shown in 5.

was gathered over a roughly 2.5 hours long mission outside of Tasmania, Australia. The dataset consists of two parts, the first consists of data from navigational sensors, in which DVL and Inertial Navigation System (INS) have been used as DR. The second part is what will be considered the ground truth, which contains the first set in combination with a visual SLAM by Mahon *et al.* [22] and USBL. The ASV, buoys and distance measurements are simulated. The noise in the simulated distance measurements has a Gaussian distribution with a σ between 0.8 - 6.0 meters, the lower value is based on the results in [23]. The static transmission times within the TDMA slot are 0, 10 and 19 seconds. The different scenarios evaluated (which can be seen in Fig. 8) are a single buoy in the centre of the AUV's survey, three buoys surrounding the survey (transmitting in a round robin fashion), an ASV performing a lawnmower survey, circling the area and a kinodynamic vehicle follower [24]. The average error on the AUV is reduced in all simulated cases as seen in Table II. In all cases the transmitting platforms have no prior knowledge about the AUV. It is adapting based on the incoming acoustic messages containing the AUV's estimated position, heading and velocity and when the localisation method is EKF, a 2x2 covariance matrix is included. It can be seen that the scenarios where the relative direction between the vehicles change is small (as single buoy, circle and lawnmower) the resulting error using NLS is large but the proposed method of choosing ToL has a slightly smaller error. This agrees with the arguments (and (7)) that the angles should be as evenly spread as possible to find a good solution, while in this case the relative motion between the vehicles does not change the relative direction between them by much. In other cases, such as the following behaviour from [24] (example of a path seen in Fig. 8), there is a larger variation between the two vehicles and both EKF and NLS are able to achieve a significant reduction in error by selection of ToL compared to static times. While using EKF, the average area and the length of the semi-major axis of uncertainty (1 std) is reduced by proposed method which can be seen in Table III.

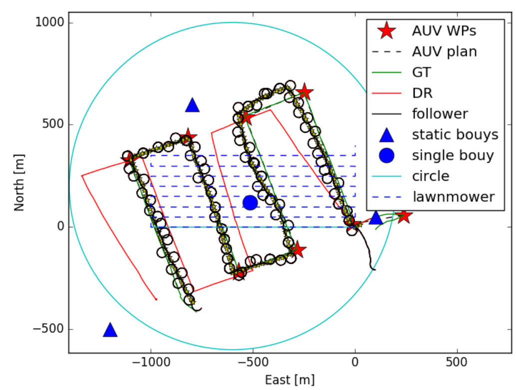


Fig. 8: Example of different paths and configurations in the evaluated scenario. In follower, the transmitting vehicle is trying to minimise the average distance between the vehicles based on [24].

V. Conclusion

In this paper we present an approach and comparison on when to to transmit acoustic localisation messages from a pre-planned path. The work looks at this from two different localisation methods: Extended Kalman Filter and trilateration solved by Non-linear Least Squares. The purpose of this is to reduce error and/or uncertainty, which the proposed method does by estimating the geometrical relationships between transmitters and receivers to find transmission times that are more beneficial than static time slots within a TDMA. The proposed method is compared to static times within the TDMA slot, and shows a reduction in each scenario for both error and uncertainty using a dataset collected by an AUV.

	NLS					EKF				
ToL	Follow	1 Buoy	3 Buoys	Circle	lawnmower	Follow	1 Buoy	3 Buoys	Circle	lawnmower
Adaptive	32.5231	52.9168	21.4256	95.3472	64.3936	12.8678	24.8505	15.0826	23.3604	25.0183
First	35.0191	67.3370	21.9328	96.3762	72.2156	17.2557	25.7671	16.1390	23.8536	25.5230
Mid	35.0186	62.4183	21.9237	97.5743	72.2774	18.8510	25.9981	16.1619	24.3698	25.2221
Last	34.9383	77.3339	21.9138	96.5713	69.7508	17.2540	25.9215	16.1406	24.3449	25.5401

TABLE II: The average error (in metres) on the AUV showing that the proposed method of selection of ToL reduces the error at all scenarios. The Dead Reckoning is 189.6065. The different paths and configurations can be seen in Fig. 8.

	EKF avg. area					EKF avg. max semi-major axis length				
ToL	Follow	1 Buoy	3 Buoys	Circle	lawnmower	Follow	1 Buoy	3 Buoys	Circle	lawnmower
Adaptive	5.6815	12.0044	14.7607	9.8183	11.0378	1.6975	3.0766	3.2370	2.7523	2.7401
First	6.3424	12.0280	15.2291	10.1190	11.1660	1.8795	3.0774	3.3412	2.8505	2.8180
Mid	6.6622	12.0943	15.1910	10.0466	11.0488	1.9639	3.0952	3.3413	2.8379	2.7856
Last	6.3430	12.0354	15.2305	10.1232	11.1685	1.8797	3.0801	3.3416	2.8518	2.8186

TABLE III: The average uncertainty (area and length of semi-major axis) represented with 1 standard deviation from the same scenarios as in Table II.

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